Harmonic Interaction Analysis in Grid Connected Converter using Harmonic State Space (HSS) Modeling

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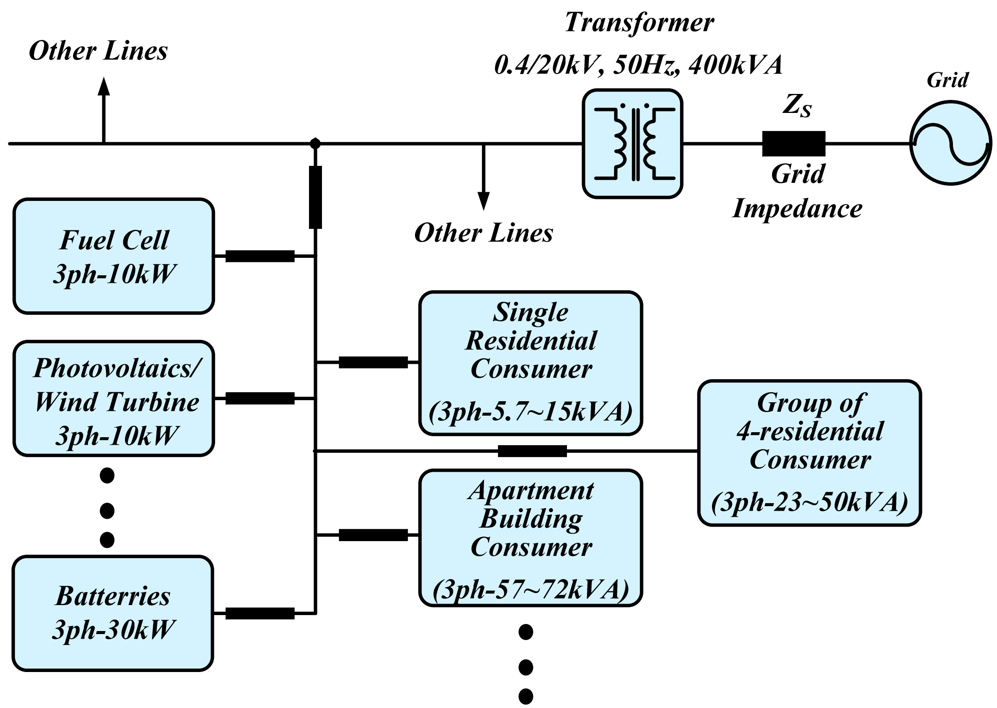
APEC

Outline

- Problem statement & Motivation
  - Time Varying Elements
- Harmonic State Space(HSS) Modeling
  - Introduction of Linear Time Periodically Varying (LTP) Theory
  - Modeling Procedure of 3 phase Grid Connected Converters
  - Harmonic Interaction analysis
- Simulation & Experimental Results
- Conclusion & Future Work

Background

- Power electronics based power system (Example)

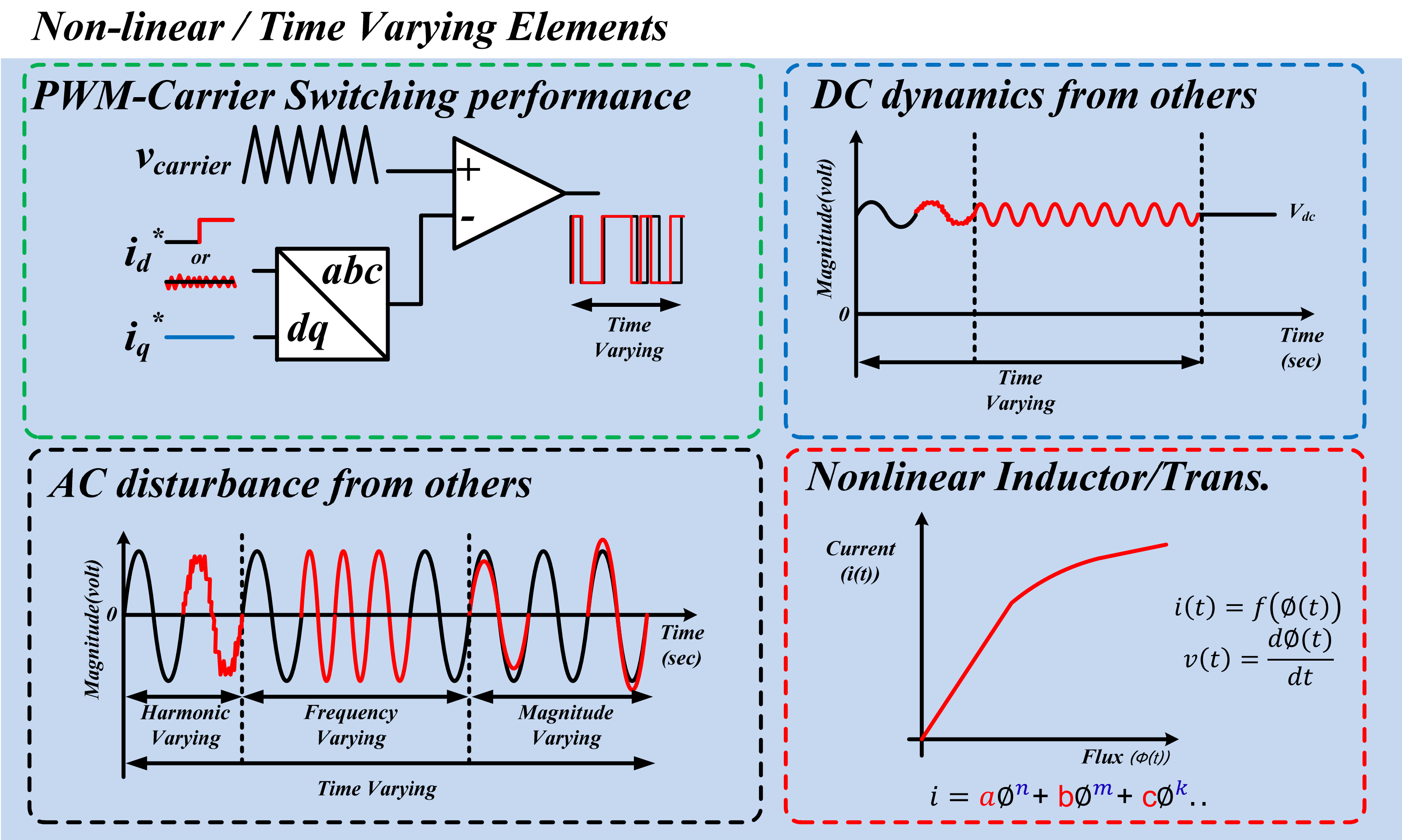


Simplified LV micro grid network (Cigre Benchmark)

- *Power electronics technology is being widely used for power generation, generation and Bidirectional power flow is changing grid impedance continuously.*

Problem Statement & Motivation (cont.)

- Time Varying Components in Power Electronics Converters

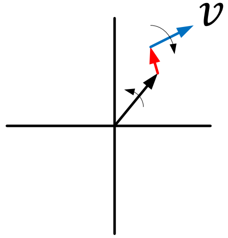


Harmonic State Space Modeling

- Introduction of LTP Theory (in electrical systems)

- Nonlinear State Space

$$\dot{x} = f(x, t, u)$$
$$y = g(x, t, u)$$

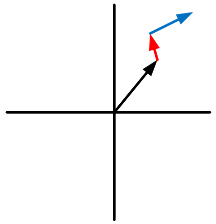


$v = v(\theta)$

- Linear Time Invariant (LTI)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Linearize about a single state

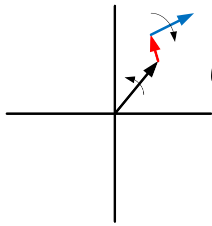


$v = constant$

- Linear Time Periodic (LTP)

$$\dot{x} = A(t)x + B(t)u$$
$$y = C(t)x + D(t)u$$

Linearize about a periodic state

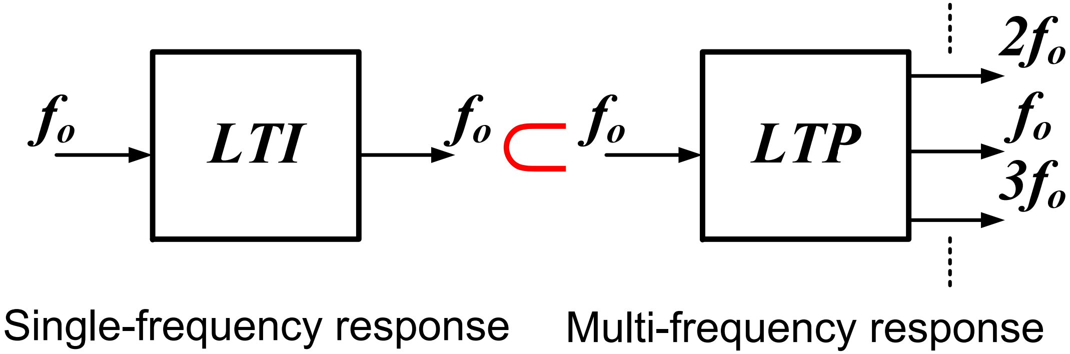


$v = v(\theta)$

$\theta = \omega t$

[1] Frequency-Domain System Identification for linear time periodic systems with application to wind turbine dynamics and CSLDV, University of Wisconsin Madison, by Dr Matthew S. Allen

❑ Introduction of LTP theory



Advantages of HSS

- **Dynamics of each harmonics**
- **Coupling analysis of AC-DC system.**
- **Including time - varying component**
- **Easy to be connected with other system matrix**
- **Possibility to figure out the characteristic, which can not be found in LTI model.**

N. M. Wereley and S. R. Hall, "Frequency response of linear time periodic systems," in *Proceedings of the 29th IEEE Conference on Decision and Control, 1990.*, 1990, pp. 3650-3655 vol.6.

Harmonic State Space Modeling

- Modeling Procedure of 3 phase Grid Connected Converters

LTI to LTP

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$
$$y(t) = Cx(t) + Du(t)$$

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad (2)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

Time Varying Fourier Coefficient

$$x(t) = \Gamma(\mathrm{t})X \qquad (3)$$

where,

$$\Gamma(\mathrm{t}) = \left[e^{-jh\omega_0 t} \cdots e^{-j2\omega_0 t}, e^{-j\omega_0 t}, 1, e^{j\omega_0 t}, e^{j2\omega_0 t} \cdots e^{jh\omega_0 t}\right]$$
$$X = [X_{-h}(t) \cdots X_{-1}(t) X_0(t) X_1(t) \cdots X_h(t)]^{\mathrm{T}}$$

Time Varying Fourier Coefficient

$$\dot{x}(t) = \dot{\Gamma}(t)X + \Gamma(t)\dot{X} \qquad (4)$$

$$\mathrm{sX}(\omega, \mathrm{t}) = \mathrm{A}(\omega) \otimes \mathrm{X}(\omega, \mathrm{t}) + \mathrm{B}(\omega) \otimes \mathrm{U}(\omega, \mathrm{t}) \qquad (5)$$
$$\mathrm{Y}(\omega, \mathrm{t}) = \mathrm{C}(\omega) \otimes \mathrm{X}(\omega, \mathrm{t}) + \mathrm{D}(\omega) \otimes \mathrm{U}(\omega, \mathrm{t})$$

Harmonic State Space Model

$$(\mathrm{s} + \mathrm{jm}\omega_0)\mathrm{X_n} = \sum_{-\infty}^{\infty} A_{n-m}X_m + \sum_{-\infty}^{\infty} B_{n-m}U_m \qquad (6)$$
$$Y_n = \sum_{-\infty}^{\infty} C_{n-m}X_m + \sum_{-\infty}^{\infty} D_{n-m}U_m$$

$$\mathrm{sX} = (\mathrm{A} - N)X + BU \qquad (7)$$
$$\mathrm{Y} = \mathrm{C}X + DU$$

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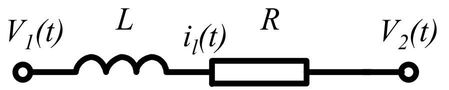
Harmonic State Space Modeling

❑ Modeling Procedure of 3 phase Grid Connected Converters

- HSS – Basic Model design (Linear - RLC circuit)

(a)



L-R Series Circuit

$$X = [\cdots \quad I_{l\_1} \quad I_{l,+0} \quad I_{l,+1} \quad \cdots]^T$$

$$Y = [\cdots \quad I_{l\_1} \quad I_{l,+0} \quad I_{l,+1} \quad \cdots]^T$$

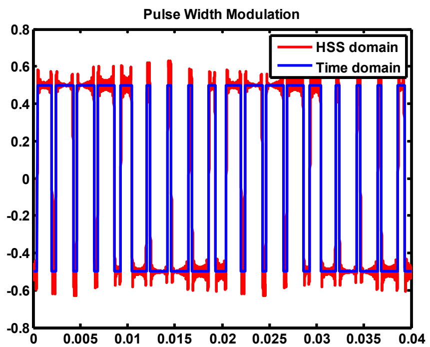
$$U = [[\cdots \quad V_{1\_1} \quad V_{1,+0} \quad V_{1,+1} \quad \cdots] \quad [\cdots \quad V_{2\_1} \quad V_{2,+0} \quad V_{2,+1} \quad \cdots]]^T$$

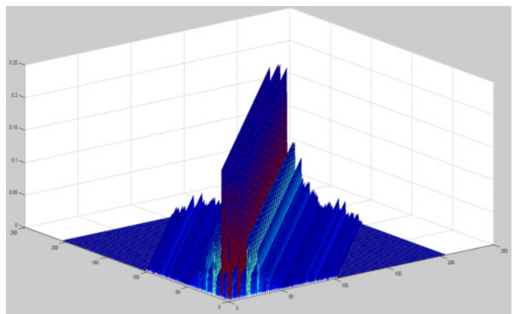
$$A = \begin{pmatrix} -\frac{R}{L} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & -\frac{R}{L} \end{pmatrix} - \begin{pmatrix} -jh\omega_0 & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & +jh\omega_0 \end{pmatrix} \quad B = \begin{pmatrix} \frac{1}{L} & \cdots & & \frac{-1}{L} & \cdots & \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ & \cdots & \frac{1}{L} & & \cdots & \frac{-1}{L} \end{pmatrix} \quad C = \begin{pmatrix} 1 & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & 1 \end{pmatrix} \quad D = \begin{pmatrix} 0 & \cdots & & 0 & \cdots & \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ & \cdots & 0 & & \cdots & 0 \end{pmatrix}$$

- HSS – Basic Model design (Switching Component)

$$\mathrm{V_{out}}(t) = s(t)V_{in}(t) \qquad (8)$$

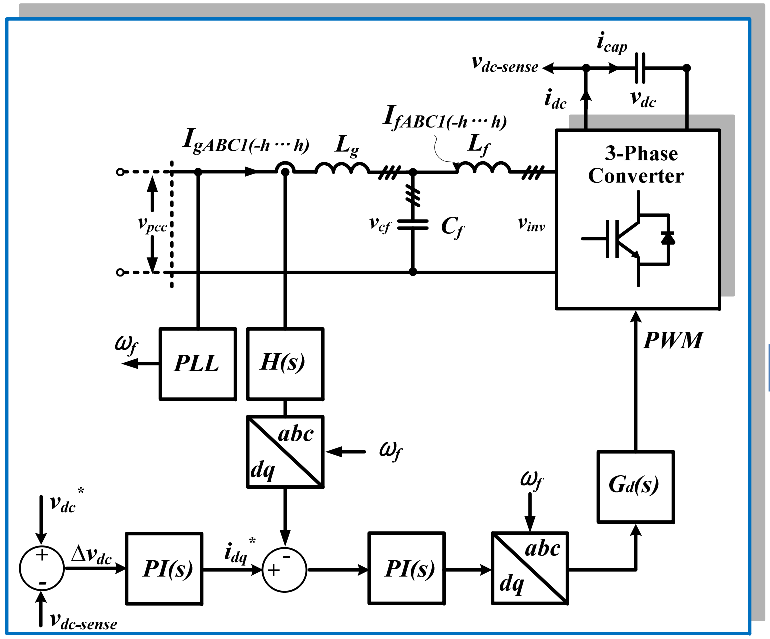
$$\begin{bmatrix} \vdots \\ \mathrm{V_{out0}} \\ \vdots \end{bmatrix} = \begin{bmatrix} S_0 & S_{-1} & \ddots \\ S_1 & S_0 & S_{-1} \\ \ddots & S_1 & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \mathrm{V_{in0}} \\ \vdots \end{bmatrix} \qquad (9)$$



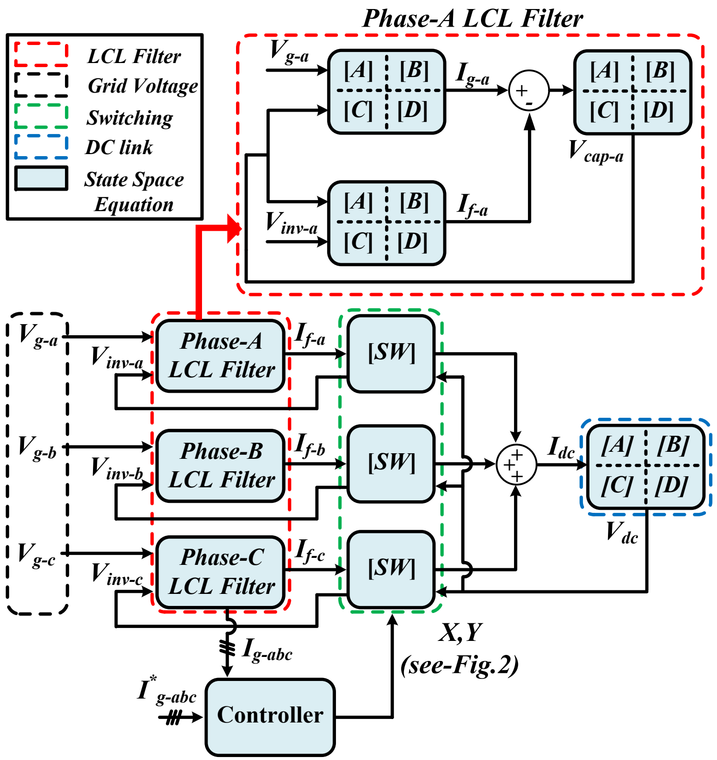


Harmonic State Space Modeling

- Modeling Procedure of 3 phase Grid Connected Converters



*Circuit diagram of the 3-phase grid connected converter with *LCL*-filter



*Block diagram of HSS modeling for 3-phase grid connected converter with *LCL*-filter

Harmonic State Space Modeling

- Modeling Procedure of 3 phase Grid Connected Converters

Switching Part

$$\mathrm{sw}(\mathrm{t}) = [s_{ab}(t) \quad s_{bc}(t) \quad s_{ca}(t)] \quad (10)$$

$$v_{inv}(t) = sw(t)^T [v_{dc}(t)] \quad (11)$$

$$i_{dc}(t) = sw(t)[i_{ga}(t) \quad i_{gb}(t) \quad i_{gc}(t)]^T \quad (12)$$

Filter part

$$\mathrm{C}_{\mathrm{dc}} \frac{dv_{dc}(t)}{dt} = i_{dc}(t) \quad (13)$$

$$v_{g-abc}(t) - v_{cap-abc}(t) = L_g \frac{di_{g-abc}(t)}{dt} + R_g i_{g-abc}(t) \quad (14)$$

$$v_{cap-abc}(t) - v_{inv-abc}(t) = L_f \frac{di_{f-abc}(t)}{dt} + R_f i_{f-abc}(t) \quad (15)$$

$$\mathrm{i}_{\mathrm{g-abc}}(t) - i_{f-abc}(t) = C_f \frac{dv_{cap-abc}(t)}{dt} \quad (16)$$

Harmonic State Space Modeling

- Modeling Procedure of 3 phase Grid Connected Converters

LTI

$$\begin{bmatrix} \dot{i}_{g-abc}(t) \\ \dot{i}_{f-abc}(t) \\ \dot{v}_{cap-abc}(t) \\ \dot{v}_{dc}(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_g}{L_g} & 0 & -\frac{1}{L_g} & 0 \\ 0 & -\frac{R_f}{L_f} & \frac{1}{L_f} & -\frac{sw(t)^T}{L_f} \\ \frac{1}{C_f} & -\frac{1}{C_f} & 0 & 0 \\ 0 & \frac{sw(t)}{C_{dc}} & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{g-abc}(t) \\ i_{f-abc}(t) \\ v_{cap-abc}(t) \\ v_{dc}(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_g} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{g-abc}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

HSS

$$\begin{bmatrix} \dot{I}_{g-abc}(t) \\ \dot{I}_{f-abc}(t) \\ \dot{V}_{cap-abc}(t) \\ \dot{V}_{dc}(t) \end{bmatrix} = \begin{bmatrix} \frac{-R_g}{L_g} I - N & 0 & \frac{-1}{L_g} I & 0 \\ 0 & \frac{-R_f}{L_f} I - N & \frac{1}{L_f} I & -\frac{\Gamma[SW^T]}{L_f} \\ \frac{1}{C_f} I & \frac{-1}{C_f} I & -N & 0 \\ 0 & \frac{\Gamma[SW]}{C_{dc}} & 0 & -N \end{bmatrix} \begin{bmatrix} I_{g-abc}(t) \\ I_{f-abc}(t) \\ V_{cap-abc}(t) \\ V_{dc}(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_g} I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{g-abc}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(18)

Harmonic State Space Modeling

- Harmonic Interaction Analysis

  - Dynamic Harmonic interaction

$$\mathrm{H_k}(s) = \sum_{l} \hat{C}_{k-l}\left((s + jl\omega_0)I - \hat{A}\right)^{-1}\hat{B}_l + D_k \qquad (s \neq 0)$$

$$\mathrm{Y(s)} = \sum_{k=-\infty}^{\infty} H_k(s - jk\omega_0)U(s - jk\omega_0)$$

where,

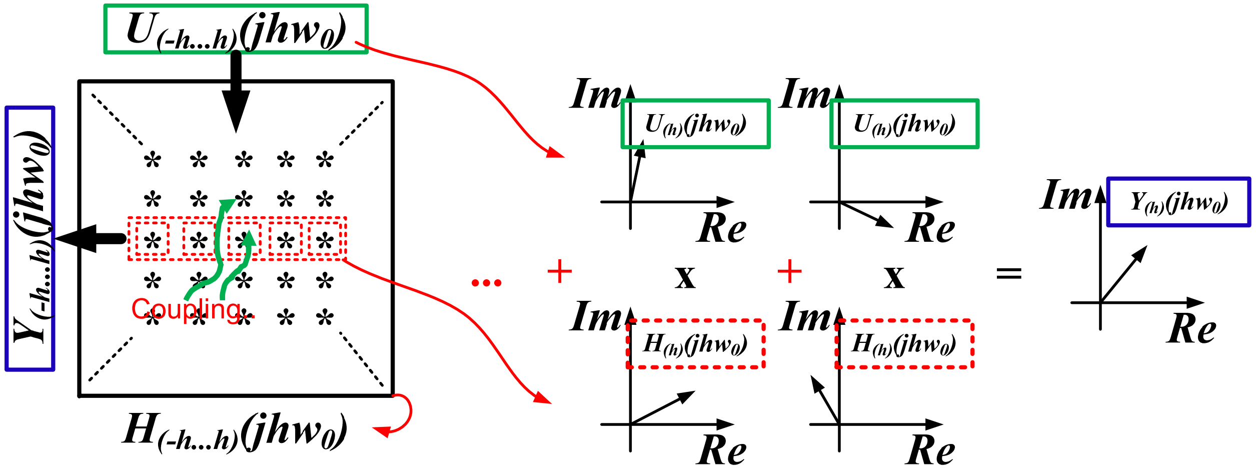
$$\mathrm{H(s)} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \\ \cdots & H_0(s - j\omega_0) & H_{-1}(s) & H_{-2}(s + j\omega_0) & \cdots \\ \cdots & H_1(s - j\omega_0) & H_0(s) & H_{-1}(s + j\omega_0) & \cdots \\ \cdots & H_2(s - j\omega_0) & H_1(s) & H_0(s + j\omega_0) & \cdots \\ & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Harmonic State Space Modeling

- Harmonic Interaction Analysis

  - Steady-state Harmonic coupling

$$\mathrm{H_k}(s) = \sum_{l} \hat{C}_{k-l}\left((s + jl\omega_0)I - \hat{A}\right)^{-1} \hat{B}_l + D_k \qquad (s = 0)$$



- Conventional approach

$$I_{th}(h) \approx \frac{V_{th}(h)}{Z_{th}(h)}$$

- Harmonic State Space approach

$$I_{th}(h) = \frac{V_{th}(h)}{Z_{th}(h)} + \frac{V_{th}(h)}{Z_{th}(h)} + \frac{V_{th}(h)}{Z_{th}(h)} + \frac{V_{th}(h)}{Z_{th}(h)} \ldots$$

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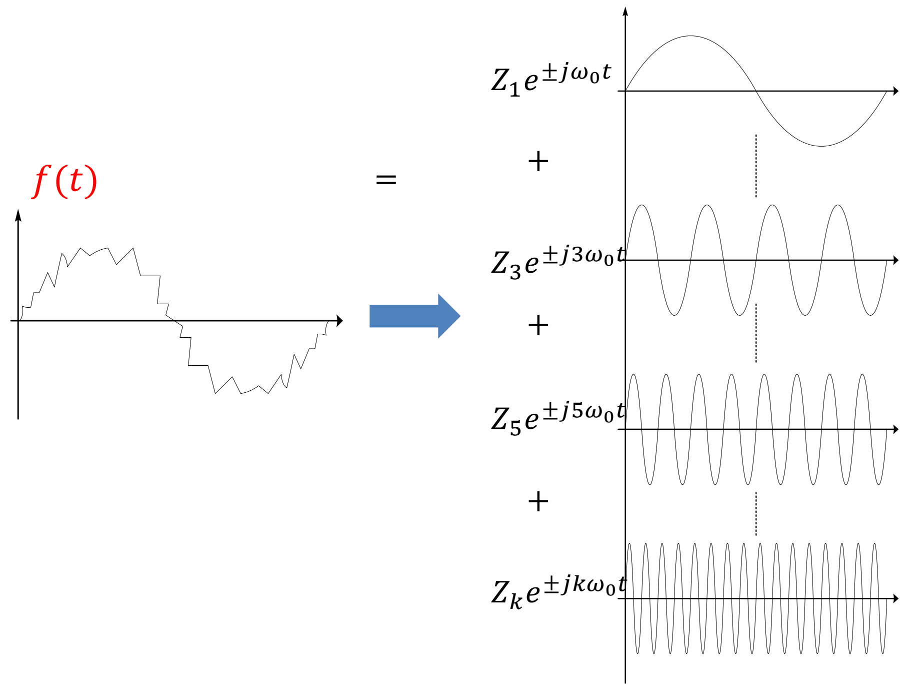
Harmonic State Space Modeling

- Harmonic Interaction Analysis
  - Time-domain simulation

$$f(t) = \sum_{k \in Z} Z_k e^{jk\omega_0 t}$$

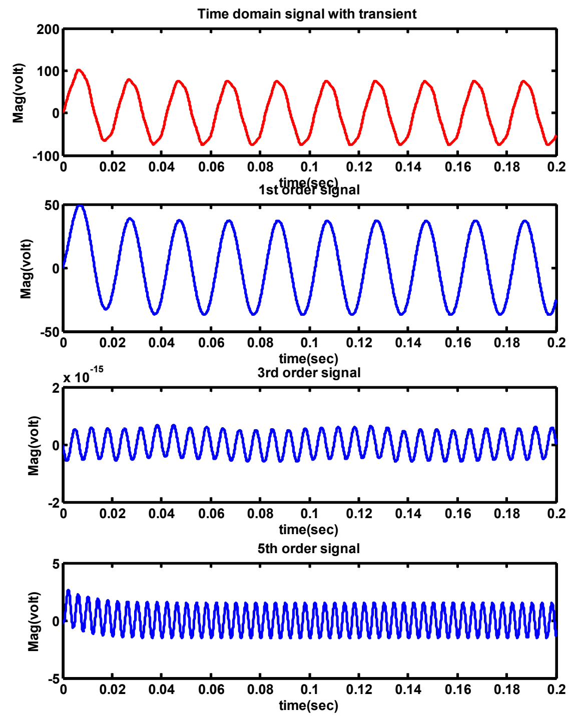
$if, s = 0$, fixed periodic signal



$$f(t) = e^{st} \sum_{k \in Z} Z_k e^{jk\omega_0 t}$$

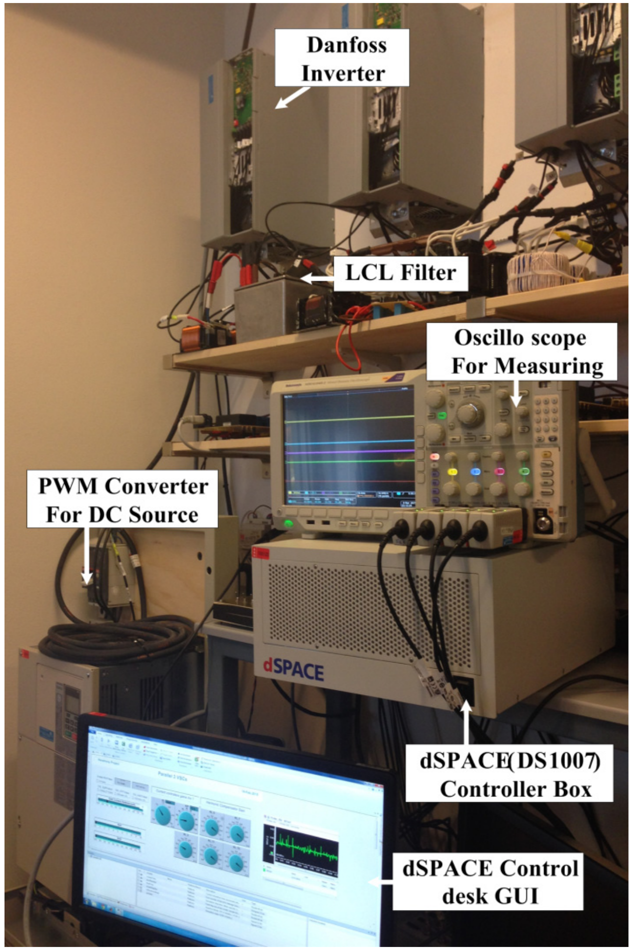
$if, s \neq 0$, dynamically time varying signals

$f(t)$ = $Z_1 e^{(s \pm j\omega_0)t}$ + $Z_3 e^{(s \pm j3\omega_0)t}$ + $Z_5 e^{(s \pm j5\omega_0)t}$ + $Z_k e^{(s \pm jk\omega_0)t}$



Harmonic State Space Modeling

- Simulation and Experimental Verification



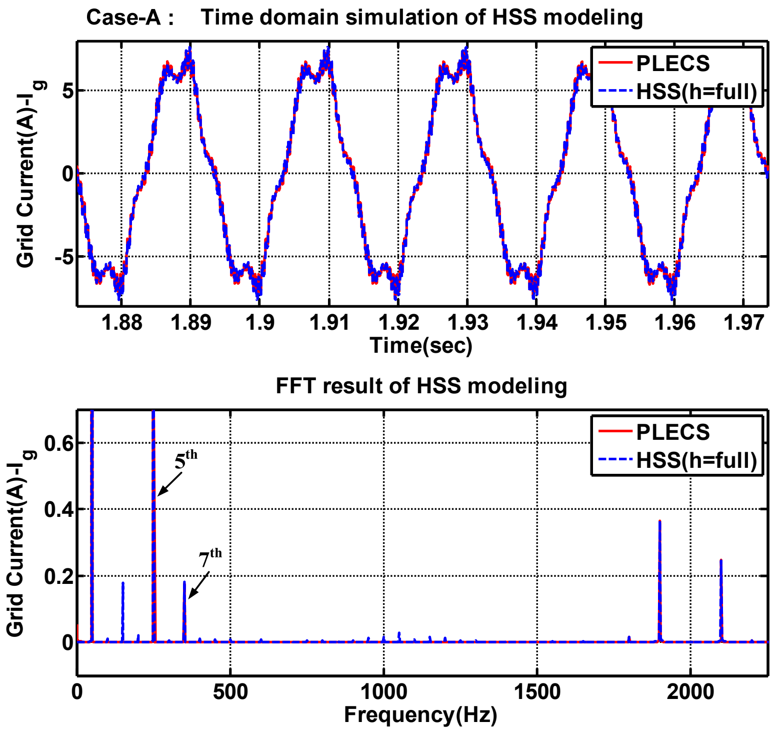
Main Parameters

Power rating	*3 kW*
L_f	*6.25 mH*
L_g	*3.3 mH*
C_f	*9.4 uF*
C_{dc}	*1000 uF*
V_{dc}	*750*
f_{sw}	*2 kHz*
-h..h	$-40^{th} \sim 40^{th}$

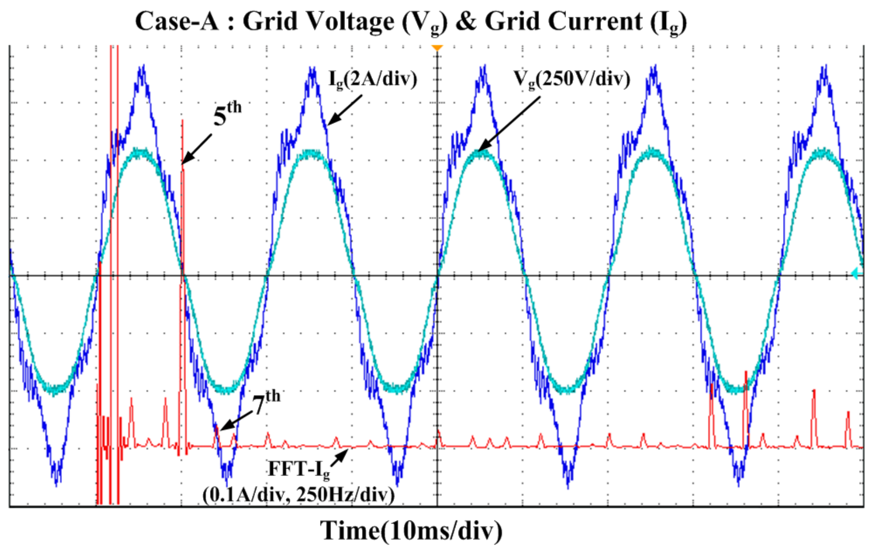
Grid condition

Grid voltage (h)	*Case A*	*Case B*
2nd	*-*	*0.2%*
5th	*4.5%*	*2.5%*
7th	*1.5%*	*4.5%*

❑ Simulation and Experimental Verification



(a)

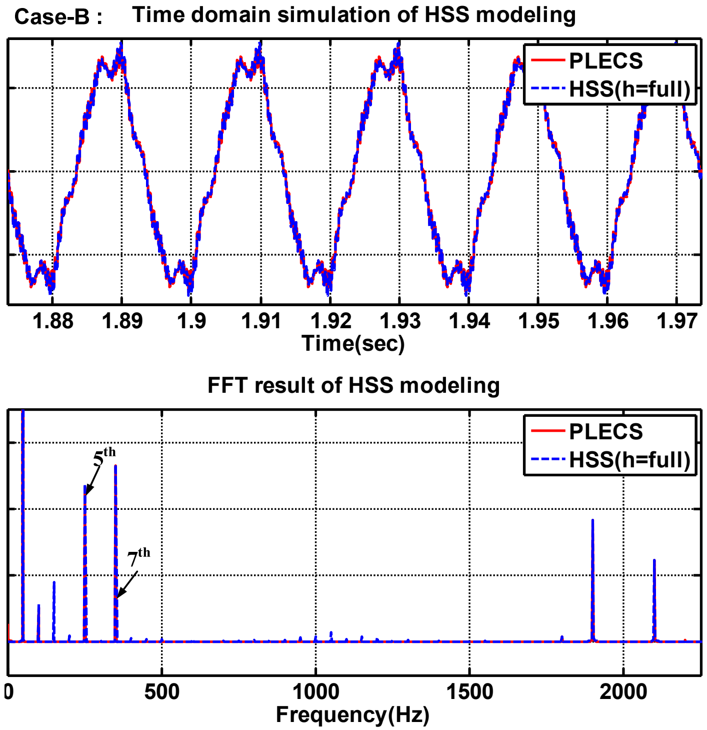


(b)

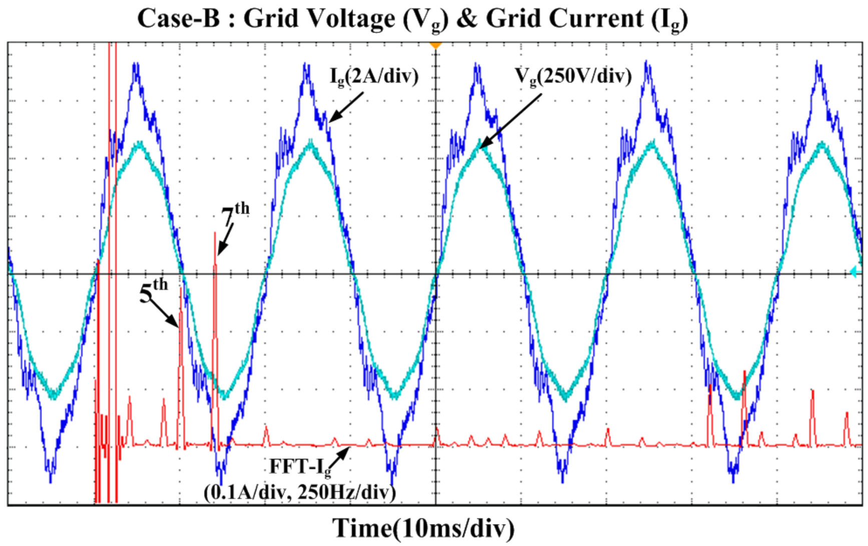
***PLECS & HSS Simulation results**
(a) Case-A (blue = grid side current, cyan=grid voltage, red = FFT waveform of grid side current)

*** Grid side inductor current experiment waveform from distorted grid voltage**
(b) Case-A (blue = grid side current, cyan=grid voltage, red = FFT waveform of grid side current)

- Simulation and Experimental Verification



(c)



(d)

***PLECS & HSS Simulation results**
(c) Case-B (blue = grid side current, cyan=grid voltage, red = FFT waveform of grid side current)

*** Grid side inductor current experiment waveform from distorted grid voltage**
(d) Case-B (blue = grid side current, cyan=grid voltage, red = FFT waveform of grid side current)

Conclusion

- *Grid connected inverter model using HSS modeling is developed.*
- *By means of HSS modeling, the time varying elements of converters can be considered and analyzed together in one domain.*
- *This model shows intuitively the harmonic coupling points of the models.*
- *The developed model will be combined with other converters for the analysis of harmonics in a large network.*
- *The non-linear passive components will be included in the model to analyze the harmonics generated in the saturation.*

Thank You! Questions?

“ THE HIDDEN HARMONY IS BETTER THAN THE OBVIOUS ”

- P. PICASSO

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